

## The Pigeon Hole Principle

**Problem 1** (*Old Problem*). A drawer contains 7 red socks, 8 blue socks, and 10 black socks. It is dark, and you can't tell what color any of the socks is. What is the smallest number of socks you can pull out of the drawer to guarantee that you get a pair of matching socks?

**Problem 2.** The population of King county is about 1,800,000 people. If humans have no more than one million hairs on their heads, show that there must be two people in the county with exactly the same number of hairs on their heads.

**Problem 3.** Given any six numbers from 1 to 10, show that some pair has an odd sum.

**Problem 4.** Show that in any group of five people, there are two people who have the same number of friends with in the group. What if there are ten people in the group?

**Problem 5.** Thirteen crates of apples arrives at a warehouse. Each crate contains apples of one of three varieties: Delicious, Granny Smith, or Pink Lady. Are there four crates of one variety? Why? Are there five crates of one variety? Why?

**Problem 6.** The integers 1, 2, ..., 10 are written on a circle, in any order. Show that there are three adjacent numbers whose sum is 18 or greater.

**Problem 7.** In chess, a king can attack any piece in a neighboring square, either vertically, horizontally, or diagonally. How many kings can you put on a  $8 \times 8$  chessboard so that no king can attack any other? Why can't you put any more than that?

**Problem 8 (Hard).** Arrange the words HEN, HOT, WIT, SAW, CAR, CUB, MOB, DIM, RED, and SUN in a circle so that any adjacent words share a letter. Can you do the same thing with HEN, HUT, WIT, SAW, CAR, CUB, MOB, DIM, RED, and SON?